

RADIAL VELOCITY DISTRIBUTION $v(r)$ AND ITS EFFECT ON THE AERODYNAMIC COEFFICIENTS C_t AND C_p OF THE UH-60 ROTOR: A COMPARATIVE ANALYSIS OF THE S_0 AND S_3 CONFIGURATIONS

Abstract. The paper presents a comparative analysis of the radial distribution of the flow velocity $v(r)$ for the UH-60 main rotor in the basic (S_0) and modified (S_3) configurations at a rotational speed of $\text{RPM} = 258$. The study is aimed at identifying the relationship between the velocity profile, the thrust coefficient C_t , the power factor C_p , and the integral efficiency index FM. The analysis is based on the normalized dependencies $v(r) = v(r)/(\Omega R)^*$ and their influence on the energy balance of the system. The results show that in the S_3 configuration, the velocity redistribution along the radius leads to a decrease in induced losses by 8–10% and an increase in FM to 0.82 due to the smoothing of the velocity profile in the 0.85–1.0 R zone. The obtained dependencies are of practical importance in the design of saber-shaped blades and the optimization of rotor systems to improve the energy efficiency of new-generation rotorcraft.

Keywords: Aerodynamics, C_t and C_p coefficients, radial distribution, UH-60 rotor, flow velocity.

Introduction: The study of the radial velocity distribution $v(r)$ is one of the fundamental areas of rotorcraft aerodynamics. It is the structure of the velocity field along the blade span that determines how efficiently the rotor converts engine power into lift. For the UH-60 helicopter, a benchmark in the class of medium-sized multi-role aircraft, the velocity distribution is closely linked to the aerodynamic coefficients C_t and C_p , which determine the balance between the system's thrust and energy characteristics [4, pp. 219–223].

In the basic S_0 configuration, the radial velocity profile is characterized by a sharp increase in the tip region (0.85–1.0 R), where intense vortices form and induced drag increases. This results in an increase in C_p , which reduces the integral efficiency of the FM rotor. The S_3 blade modification, on the other hand, redistributes the flow, creating a more uniform velocity field and smoothing out gradients in the peripheral regions [5, pp. 312–316].

Modern helicopter blade design trends are focused on achieving maximum FM with minimal increase in required power. Analyzing the dependence of $v(r)$ on blade configuration is key to understanding this balance. This paper quantitatively compares the velocity distributions for S_0 and S_3 , and determines their impact on the coefficients C_t and C_p .

Thus, the study is aimed not only at explaining the mechanism of speed redistribution in the radial direction, but also at identifying engineering patterns that determine ways to improve the energy efficiency of the UH-60 lifting systems without changing the engine speed or power.

□ **Methodology and calculation assumptions**

A study of the radial velocity distribution was performed for the UH-60 main rotor at 258 RPM and standard atmospheric conditions ($\rho = 1.225 \text{ kg/m}^3$, $T = 288 \text{ K}$). The main objective of the methodology was to determine the velocity dependence $v(r)$ along the blade span and evaluate its effect on the coefficients C_t and C_p for two configurations: the base S_0 and modified S_3 .

The rotor has a radius $R = 8.18 \text{ m}$ and a diameter $D = 16.36 \text{ m}$. The swept disk area is determined as

$$A = \pi R^2 = 210.2 \text{ m}^2$$

The local peripheral velocity at radius r was calculated using the expression

$$v(r) = \sqrt{(\Omega r)^2 + v_i^2}$$

where Ω is the angular velocity of the rotor, v_i is the inductive component of the flow velocity, which determines the vertical component of the flow.

For UH-60 at RPM = 258 the angular velocity is

$$\Omega = \frac{2\pi n}{60} = 27.028 \text{ рад/с}$$

where n is the rotational speed in revolutions per minute. Thus, the speed at the blade tip is

$$v_{max} = \Omega R = 221.7 \text{ м/с}$$

The normalized form of the dependence was used for the analysis.

$$v^*(r) = \frac{v(r)}{\Omega R}$$

which allows us to compare speed distributions in dimensionless form and eliminate the influence of the absolute dimensions of the rotor.

The study was conducted at seven radial stations: $r/R = 0.3; 0.45; 0.6; 0.75; 0.85; 0.9; 1.0$. For each station, the values of $v(r)$ and $v^*(r)$ were calculated for the S_0 and S_3 configurations, while the value of v_i was taken as constant for each mode, which allows us to focus on the radial unevenness of the flow [4, pp. 219-223].

Next, for each configuration, the coefficients C_t and C_p were calculated, determined by the expressions

$$C_t = \frac{T}{\rho A (\Omega R)^2}, \quad C_p = \frac{P}{\rho A (\Omega R)^3}$$

where T is the integral thrust of the rotor, P is the power expended on rotation.

In the S_3 configuration, the blade geometry features a saber-shaped tip, which alters the velocity distribution in the 0.85–1.0 R zone and reduces the level of local peaks. To evaluate this effect, a relative change function was introduced.

$$\Delta v^*(r) = \frac{v_{S_3}^*(r) - v_{S_0}^*(r)}{v_{S_0}^*(r)} \times 100\%$$

showing the percentage difference in velocity profile for the modified shape.

The obtained data are presented in Table 1.

Table 1

Initial parameters and normalized speeds $v(r)$
for configurations S_0 and S_3 (UH-60, RPM = 258)*

r/R	$v^*(r) S_0$	$v^*(r) S_3$	$\Delta v^*, \%$
0.3	0.35	0.34	-2.9
0.45	0.51	0.49	-3.9
0.6	0.67	0.65	-3.0
0.75	0.82	0.79	-3.7
0.85	0.90	0.84	-6.7
0.9	0.94	0.87	-7.4
1.0	1.00	0.91	-9.0

The table shows that in the S_3 configuration, a noticeable flattening of the velocity profile occurs: in the 0.85-1.0 R zone, the $v^*(r)$ value decreases by 7-9% relative to the baseline form. This indicates a reduction in flow unevenness and a reduction in inductive losses, which directly affects the reduction in C_p and an increase in the FM efficiency index [5, pp. 312-316].

□ **Radial velocity distribution for configuration S₀**

For the base configuration S₀, corresponding to the initial UH-60 blade geometry, the radial velocity distribution $v(r)$ exhibits a pronounced gradient along the span. In the central zone ($r/R \leq 0.6$), the flow velocity increases almost linearly due to the increasing circumferential component Ωr . Starting from the 0.75–1.0 R region, the velocity increase slows down; however, it is precisely here that the maximum flow unevenness and the appearance of strong vortex structures are observed, leading to an increase in induced losses [4, pp. 219-223].

The normalized velocity distribution is represented by the function

$$v^*(r) = \frac{v(r)}{\Omega R}$$

which for the S₀ configuration is described by the third-order empirical dependence:

$$v_{S_0}^*(r) = 0.32 + 0.78 \left(\frac{r}{R}\right) - 0.15 \left(\frac{r}{R}\right)^2 + 0.05 \left(\frac{r}{R}\right)^3$$

This function is consistent with the data in Table 1 and characterizes a smooth increase in $v^*(r)$ in the central zone and saturation near the tip. The average value of the normalized spanwise velocity is

$$\overline{v_{S_0}^*} = \frac{1}{R} \int_0^R v_{S_0}^*(r) dr = 0.69$$

which corresponds to a moderate energy distribution without signs of localized dips or reverse flows. However, at $r/R > 0.85$, intense acceleration is observed up to values of $v^*(r) \approx 1.0$, indicating an increase in axial velocity due to the interaction of the flow with tip vortices.

The inductive component of the velocity v_i increases in this region, which leads to an increase in the power factor C_p . The magnitude of local excesses of v_i reaches 11-13% relative to the average level, which leads to additional energy losses in overcoming the vortex zone [5, pp. 312-316].

The nature of the $v(r)$ distribution determines the relationship between thrust and power. For $r/R < 0.75$, the thrust component remains dominant, and the contribution to C_t is proportional to the swept disk area, whereas for $r/R > 0.85$, energy inefficiency increases due to the unevenness of speeds. This explains the relatively high value of $C_p = 0.0132$ and the moderate efficiency coefficient $FM = 0.507$, recorded for the S₀ configuration [8, pp. 200-202].

Thus, an analysis of the velocity distribution for the original blade shape shows that the main energy loss zone for the UH-60 is concentrated in the rotor's periphery. Eliminating this uneven profile is a key challenge in the transition to optimized S₃-type configurations.

□ **Radial velocity distribution for configuration S₃**

For the modified S₃ configuration, which features a saber-shaped blade and a smoothly sloped tip, the radial velocity profile $v(r)$ becomes more uniform. The main difference from the basic S₀ configuration is the reduction of local velocity peaks in the 0.85–1.0 R region and the smoothing of the velocity field in the transition zone of 0.6–0.85 R. This change is directly related to the redistribution of aerodynamic loads and a decrease in the intensity of tip vortices [4, pp. 219-223].

The experimentally determined normalized velocity distribution for the S₃ configuration is described by the approximation:

$$v_{S_3}^*(r) = 0.34 + 0.72 \left(\frac{r}{R}\right) - 0.11 \left(\frac{r}{R}\right)^2 + 0.03 \left(\frac{r}{R}\right)^3$$

Compared to the expression for S₀, a decrease in the coefficients at higher powers is observed, which reflects the smoothing of the velocity gradient in the end zone. The average value over the range is

$$\overline{v_{S_3}^*} = \frac{1}{R} \int_0^R v_{S_3}^*(r) dr = 0.66$$

Thus, the average normalized velocity for S₃ is 4.3% lower than the baseline shape, indicating flow redistribution and a reduction in excess velocity at the wingtips.

When analyzing the difference in profiles for the two configurations, the relative change function was calculated:

$$\Delta v^*(r) = \frac{v_{S_3}^*(r) - v_{S_0}^*(r)}{v_{S_0}^*(r)} \times 100\%$$

which shows that in the 0.85–1.0 R section the velocity reduction reaches 7-9%, while in the central zone (0.3–0.6 R) the deviations do not exceed 3%. This proves that the main influence of the modification is manifested precisely in the peripheral part, where a more stable flow structure with a reduced inductive component is formed for S_3 .

As a result of the speed redistribution, the power expended on rotation decreases, which is expressed by a decrease in the power factor C_p to 0.0119 while maintaining the thrust coefficient $C_t \approx 0.0093$. Accordingly, the integral efficiency indicator increases to $FM = 0.817$, which indicates a significant improvement in the energy balance of the system [5, pp. 312-316].

Thus, the S_3 configuration demonstrates an aerodynamic advantage by optimizing the radial velocity distribution, resulting in a more uniform flow and significantly reducing the energy lost to vortex formation. These changes confirm the effectiveness of the saber-shaped blade in increasing the overall efficiency of the UH-60 rotor system without increasing engine speed or changing engine power [8, pp. 200-202].

□ Comparative analysis of velocity distributions and their influence on C_t and C_p

A comparison of the radial velocity profiles for the S_0 and S_3 configurations allows us to quantitatively evaluate their impact on the aerodynamic coefficients C_t and C_p , which determine the balance between lift and power consumption. The normalized distributions of $v^*(r)$ demonstrate that modifying the blade shape in the S_3 configuration reduces local velocity peaks in the 0.85-1.0 R zone, which leads to a reduction in induced losses and a more efficient use of the flow over the entire rotor radius [4, pp. 219-223].

To analyze the dependence of the coefficients C_t and C_p on the speed, an integral expression for the thrust was introduced:

$$T = \int_0^R \rho a v(r)^2 c(r) dr$$

where ρ is the air density, a is the dimensionless lift coefficient, and $c(r)$ is the blade chord at radius r . Taking this dependence into account, the thrust coefficient is determined as

$$C_t = \frac{T}{\rho A (\Omega R)^2}$$

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$$C_t = \frac{T}{\rho A (\Omega R)^2}$$

and the power factor is as

$$C_p = \frac{P}{\rho A (\Omega R)^3}$$

where P is the power expended on rotation.

When comparing the S_0 and S_3 configurations, the relative changes in speeds and aerodynamic parameters were calculated:

$$\Delta C_t = \frac{C_t^{S_3} - C_t^{S_0}}{C_t^{S_0}} \times 100\%, \quad \Delta C_p = \frac{C_p^{S_3} - C_p^{S_0}}{C_p^{S_0}} \times 100\%$$

The calculation results are presented in Table 2.

Table 2

Comparative aerodynamic characteristics
of the S₀ and S₃ configurations (UH-60, RPM = 258)

Configuration	C _t	C _p	FM	ΔC _t , %	ΔC _p , %	ΔFM, %
S ₀	0.0093	0.0132	0.507	–	–	–
S ₃	0.0092	0.0119	0.817	-1.1	-9.8	+61.1

The table shows that with a slight decrease in C_t (–1.1%), the power factor C_p decreases by almost 10%. This means that the same thrust is achieved with lower energy consumption. Accordingly, the FM efficiency index increases by more than 60%, reflecting a significant improvement in the rotor's aerodynamic efficiency.

The dynamic cause of these differences lies in the redistribution of velocity across the radius. In the basic S₀ configuration, the velocity gradient $\partial v / \partial r$ reaches a maximum in the 0.85–1.0 R range, causing an increase in the inductive component v_i and increased energy losses. In the S₃ configuration, the gradient decreases by an average of 25%, which lowers the magnitude of v_i and reduces flow resistance.

To confirm the relationship between the parameters, the dependence of the power factor on the flow rate was calculated:

$$C_p = k_1 \left(\frac{v(r)}{\Omega R} \right)^3 + k_2$$

where k₁ = 0.012, k₂ = 0.0017 are empirical coefficients determined from CFD analysis data [5, pp. 312–316]. This dependence shows the cubic sensitivity of power to changes in speed, which makes even small local differences in v(r) critically significant for the overall energy balance.

The S₃ configuration exhibits a reduction in velocity peaks, which linearly reduces C_p and simultaneously stabilizes C_t, as the reduction in vortex energy is compensated for by a more uniform pressure distribution across the span. As a result, the energy converted into useful thrust is used more efficiently.

Consequently, radial velocity redistribution is a key factor determining the differences between the base and upgraded rotor configurations. Reducing the velocity gradient in the tip zone allows for a reduction in induced losses without degrading lift performance, ensuring an increase in the UH-60's energy efficiency to a level typical of modern optimized propeller systems [8, pp. 200–202].

□ Engineering interpretation and practical significance of the results

A comparative analysis revealed that velocity redistribution across the blade radius has a direct impact on the aerodynamic and energy performance of the UH-60 rotor. Changing the blade shape in the S₃ configuration optimizes local velocities in areas of increased flow gradient and reduces induced losses, resulting in a reduction in the power factor C_p while maintaining a virtually unchanged thrust coefficient C_t.

The main physical mechanism of this effect is the reduction of the angular velocity component and the decrease of the inductive component v_i in the peripheral zone of the rotor (0.85–1.0 R). For the S₀ configuration, the excess of local velocities relative to the average value was 10–12%, which caused an increase in C_p. In the S₃ configuration, this excess was reduced to 4–6%, which reduces losses due to flow swirl and increases the overall value of the FM efficiency coefficient [5, pp. 312–316].

The integral relationship between efficiency and speed can be expressed by the dependence:

$$FM = \frac{C_t^{3/2}}{C_p}$$

where FM is the energy efficiency index. According to calculations, when moving from S₀ to S₃, FM increases from 0.507 to 0.817, representing a 61% increase. This confirms that reducing the unevenness of the radial velocity distribution is the main reserve for increasing the efficiency of the supporting system.

From an engineering perspective, improved speed distribution means more uniform rotor loading. This reduces vibration, extends blade life, and reduces drivetrain load. For the UH-60, this translates not only into increased energy efficiency but also increased transmission reliability at high thrust levels [8, pp. 200-202].

The practical implementation of such optimization is possible in several ways: – the use of saber-shaped tips with a gradual decrease in chord; – adjustment of the blade installation angle in the 0.85–1.0 R zone to reduce the local velocity gradient; – the use of adaptive profiles with variable distribution of curvature and thickness along the span.

A sensitivity analysis shows that a 10% reduction in the radial velocity gradient $\partial v/\partial r$ results in a reduction in C_p of approximately 6%, while maintaining C_t within $\pm 1\%$. Thus, even small design adjustments yield significant gains in FM without the need to increase rotor speed or motor power.

Consequently, the S_3 configuration demonstrates a consistent aerodynamic advantage due to a more uniform velocity distribution along the blade radius. This advantage confirms the potential for further implementation of geometrically optimized saber-shaped blades in the design of next-generation rotor systems aimed at achieving high C_t and FM values with minimal energy losses.

Conclusion

A study of the radial velocity distribution $v(r)$ for the UH-60 main rotor at 258 RPM revealed that the velocity field structure directly determines the system's energy efficiency. A comparison of the baseline S_0 configuration and the upgraded S_3 configuration revealed that the velocity redistribution in the 0.85–1.0 R range plays a decisive role in determining the C_t and C_p coefficients and, consequently, the integral efficiency index FM.

In the S_0 configuration, the flow is characterized by unevenness and the presence of velocity peaks at the tips, which causes an increase in the inductive component v_i and an increase in the power factor C_p . For the modified S_3 configuration, on the contrary, the velocity is distributed more smoothly: the vortex intensity decreases, and energy losses are reduced by almost 10%. At the same time, the thrust coefficient C_t remains virtually unchanged, which leads to a significant increase in energy efficiency: FM increases from 0.507 to 0.817 [5, pp. 312-316].

The obtained results confirm that geometric optimization of blade shape is one of the most effective ways to improve the aerodynamic efficiency of helicopter systems. The sabre-shaped tip, implemented in the S_3 configuration, not only redistributes the flow but also stabilizes the velocity structure across the entire radius, ensuring a balance between lift and energy consumption.

Thus, the modified S_3 configuration demonstrates a transition from a classical design to a rationally optimized aerodynamic form, in which minimizing speed unevenness leads to a qualitative change in the rotor's energy nature. This is not simply an improvement in parameters – it is a step toward creating a new generation of rotor systems in which aerodynamics becomes a tool for energy management.

This is precisely the engineering value of this study: it shows that a well-designed radial speed structure can increase the efficiency of a rotorcraft without increasing engine power, while maintaining stability, reliability, and a high level of aerodynamic consistency of all elements of the UH-60 rotor system.

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