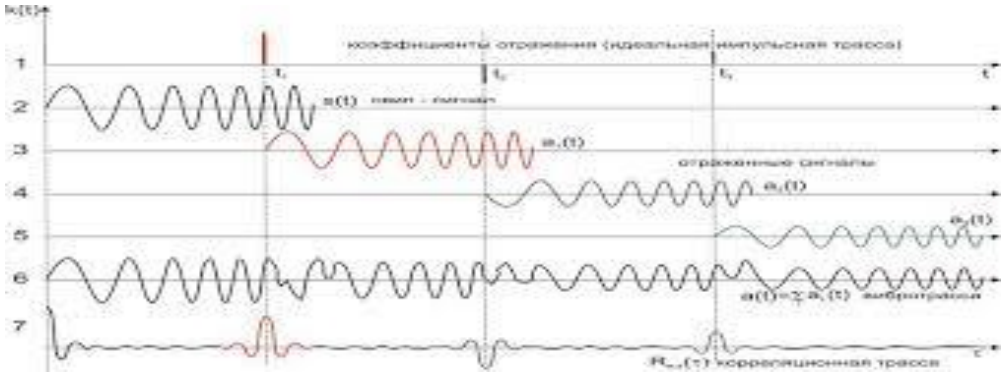
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NEW OPPORTUNITIES FOR SIGNAL EXTRACTION AND ANALYSIS. KAPTZZ-CONVERSION

Abstract: The widely known Fourier transform allows analyzing the frequency spectrum of any process. For decades, it has not essentially evolved, only scaled. Fourier transform process analysis is performed in all fields: economic, social, scientific, and applied. Growing volumes and diversity of information require more precise analysis methods and algorithms for finding subtler differences in any processes.

Keywords: Kaptz conversion, Fourier transform, signal processing, sweep signals, Kaptz spectrum.

The use of long vibration signals with a specified law of frequency change in Deep Seismic Sounding is characterized by a new approach to the initial stage of signal processing. An example of sweep signals (2-3-4-5) with increasing frequency and their sum 6.



In particular, a new analyzed parameter of vibration signals, or so-called sweep signals, has emerged, of the form $sin\,\alpha t^2$, where the frequency growth rate is - $\alpha = const \ll 1$ (Hz/sec), unlike explosive pulses, for which α→∞. Therefore, the question arises: is it possible, using an additional parameter, to increase the capabilities of extracting and analyzing vibration signals? Such an attempt was undertaken in [2], where vibration signals are represented in the spectral-time domain by calculating frequency spectra in a "sliding window" along the entire vibration signal. It is shown that there exists an optimal analysis interval size ($T = \sqrt{\frac{2\pi}{\alpha}}$), greater or smaller than which we will not obtain a better resolved spectrum. The resulting resolution in the spectral-time domain, estimated as half of the analysis interval - T/2, does not satisfy the task of isolating individual waves in a complex interference field. The spectral-time representation provides a general picture of the wave process and the analysis of nonlinear phenomena.

This work proposes a transform (let us call it the Kaptz-conversion) by which the frequency growth rate of a vibration signal can be calculated similarly to the calculation of frequency spectra. It also demonstrates the capabilities of this method for the task of extracting and analyzing vibration signals. In doing so, the theory of the Fourier transform is used.

It should be noted that the frequency growth rate of sweep signals is set instrumentally; there is no analytical calculation for it.

Let us consider a series of the form $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n cos\, nx^2 + b_n sin\, nx^2\ )$, in which the partial sums are linear combinations of functions from the system:

1, cos x^2, sin x^2, cos $2x^2$, sin $2x^2$, …cosnx^2, sin$n$ x^2 (0)

And let us assume that the function $[0, \sqrt{2\pi}]$ integrable on the interval f(x) can be expanded into this series: $$f(x) \sim a_0 + \sum_{n=1}^{\infty}(a_n cos\, nx^2\ + b_n\, sin\ \ nx^2\ ) \quad (1)$$

The trigonometric series (1) is analogous to the Fourier series [1], except that one argument is squared. To demonstrate the validity of expansion (1), we use the theory of function expansion into a Fourier series [1]. All proofs will be similar. The only specific feature in the proofs is the use of a weight function – x, which ensures the complete correspondence of expansion (1) to the Fourier series. To do this, we will prove the orthogonality of the function system (0), find the coefficients a_n, b_n , and calculate the Dirichlet integral, upon which most of the proofs are built.

This system of functions (0) will be orthogonal with the weight x and only with it (this is one of the author's know-hows). Let us show on the interval $[\varepsilon_1, \varepsilon_2]$, where $\varepsilon_1=0$, $\varepsilon_2\sqrt{2\pi}$: according to the definition of orthogonality, let us consider combinations of multiplication of functions from system (0) and integrate them with weight x:

$$\int_{\varepsilon_1}^{\varepsilon_2} x\, sinnx^2 cosmx^2 dx = \frac{1}{2}\int_{\varepsilon_1}^{\varepsilon_2} sin\, nx^2 cosmx^2 dx^2 =$$
$$=\frac{1}{4}\int_{\varepsilon_1}^{\varepsilon_2}[sin(n+m)x^2 + cos(n-m)x^2]\, dx^2 = 0 \quad (2)$$

For n≠m

$$\int_{\varepsilon_1}^{\varepsilon_2} x\, cosnx^2 cosmx^2 dx = \frac{1}{4}\int_{\varepsilon_1}^{\varepsilon_2}[cos(n+m)x^2 + cos(n-m)x^2]\, dx^2 = 0 \quad (3)$$

For n≠m

$$\int_{\varepsilon_1}^{\varepsilon_2} x\, sinnx^2 sinmx^2 dx = \frac{1}{2}\int_{\varepsilon_1}^{\varepsilon_2} sin\, nx^2 sinmx^2 dx^2$$
$$=\frac{1}{4}\int_{\varepsilon_1}^{\varepsilon_2}[cos(n-m)x^2 - cos(n+m)x^2]\, dx^2 = 0 \quad (4)$$

For m=n

$$\int_{\varepsilon_1}^{\varepsilon_2} xsinnx^2 cosmx^2 dx = \frac{1}{2}\int_{\varepsilon_1}^{\varepsilon_2} sinnx^2 cosmx^2 dx^2 =$$
$$=\frac{1}{4}\int_{\varepsilon_1}^{\varepsilon_2}[sin(n-m)x^2 + sin(m+n)x^2]\, dx^2 = 0 \quad (5)$$

$$\int_{\varepsilon_2}^{\varepsilon_1} xsin^2 nx^2\, dx = \frac{1}{2}\int_{\varepsilon_2}^{\varepsilon_1} sin^2 nx^2\, dx^2 = \frac{1}{4}\int_{\varepsilon_2}^{\varepsilon_1}[1 - cos2\pi nx^2]\, dx^2 = \frac{\pi}{2} \quad (6)$$

$$\int_{\varepsilon_2}^{\varepsilon_1} xcos^2 nx^2\, dx = \frac{1}{2}\int_{\varepsilon_2}^{\varepsilon_1} cos^2 nx^2\, dx^2 = \frac{1}{4}\int_{\varepsilon_2}^{\varepsilon_1}[1 + cos2\pi nx^2]\, dx^2 = \frac{\pi}{2} \quad (7)$$

For an arbitrary interval [0,T], the orthogonality condition with a weight holds for the system of functions:

$$1, cos\frac{\pi}{T}x^2, cos\frac{2\pi}{T}x^2, \ldots, cos\frac{n\pi}{T}x^2, sin\frac{\pi}{T}x^2, sin\frac{2\pi}{T}x^2, \ldots, sin\frac{n\pi}{T}x^2;$$

Let us determine the coefficients a_m, b_m in expansion (1) using the Euler-Fourier method. Let the function be integrable on the interval $[0,\sqrt{2\pi}]$. Suppose that this expansion holds and integrate it term by term with weight x from 0 to $\sqrt{2\pi}$, we obtain:

$$\int_0^{\sqrt{2\pi}} xf(x)dx = \pi a_0 + \sum_{n=1}^{\infty}\left[a_m\int_0^{\sqrt{2\pi}} xcosmx^2 dx + b_m\int_0^{\sqrt{2\pi}} x\, sinmx^2 dx\right] \quad (8)$$

And since:

$$\int_0^{\sqrt{2\pi}} cos\, nx^2 dx^2 = \frac{sinnx^2}{n}\Big|_0^{\sqrt{2\pi}} = 0 \quad (9)$$

$$\int_0^{\sqrt{2\pi}} cos\, nx^2 dx^2 = \frac{sinnx^2}{n}\Big|_0^{\sqrt{2\pi}} = 0; \quad (10)$$

Finally, we obtain $a_0 = \frac{1}{2\pi}\int_0^{\sqrt{2\pi}} x$*f(x)dx; (11)

To determine the coefficient a_m in series (1), let us multiply both sides of the equality by x*cosmx^2 and integrate term by term:

We obtain

$$\int_0^{\sqrt{2\pi}} x*f(x)*cosmx^2 dx = a_0*\int_0^{\sqrt{2\pi}} x*cosmx^2 dx +$$
$$+\sum_{n=1}^{\infty}\left[a_m*\int_0^{\sqrt{2\pi}} x*cosmx^2 cosnx^2 dx + b_m*\int_0^{\sqrt{2\pi}} x*sinnx^2 cosmx^2 dx\right] \ldots\ldots\ldots..(12)$$

The first term vanishes by virtue of (7). Then, using formulas (3) and (5), only the integral for n=m with the factor a_m will remain:

$$a_m = \frac{1}{2\pi}\int_0^{\sqrt{2\pi}} x*f(x)*cosmx^2 dx; \quad (13)$$

Similarly, multiplying (1) by x*sinmx^2 and integrating term by term, we obtain:

$$b_m = \frac{1}{2\pi} x * f(x) * sinmx^2 dx; \quad (14)$$

The form of the coefficients is analogous to the coefficients of the Fourier series, except that we use a weight function for the stability of the transformation.

To study the behavior of series (1) at some specific point x=x$_0$,

we construct a convenient expression for its partial sum:

$$S_n(x_0) = \frac{a_0}{2} + \sum_{n=1}^{n}(a_n cosnx_0^2 + b_n sinnx_0^2) \quad (15)$$

Let us substitute the integral expressions for the coefficients and bring cos(mx) sin(mx) under the integral sign:

$$S_n(x_0) = \frac{2}{2\pi}\int_0^{\sqrt{2\pi}} u\,f(u)du + \sum_{m=1}^{n}\frac{2}{\pi}\int_0^{\sqrt{2\pi}} u\,f(u)[cosmu^2 cosmx_0 + sinmusinmx_0^2]du =$$
$$= \frac{1}{\pi}\int_0^{\sqrt{2\pi}} f\,(u)\left\{\frac{1}{2} + \sum_{m=1}^{n} cos\,m(x-x_0)\right\}du^2 = |u^2 = x\ x_0^2 = x_0\,| =$$
$$= \frac{1}{\pi}\int_0^{2\pi} f\,(\sqrt{x})\left\{\frac{1}{2} + \sum_{m=1}^{n} cos\,m(x-x_0)\right\}dx \quad (16)$$

Using the formula to transform the expression in curly brackets, we obtain:

$$\frac{1}{2} + \sum_{m=1}^{n} cos\,m(x-x_0) = \frac{sin(2\pi+1)\frac{x-x_0}{2}}{2sin\frac{x-x_0}{2}}; \quad (17)$$

And finally:

$$S_n(x_0) = \frac{1}{\pi}\int_0^{2\pi} f\,(\sqrt{x})\frac{sin(2\pi+1)\frac{x-x_0}{2}}{2sin\frac{x-x_0}{2}}dx \quad (18)$$

This important integral bears Dirichlet's name. Furthermore, by using the lemmas of Dirichlet and Riemann criteria in partial sum (18), the convergence of series (1) is demonstrated.

Thus, we have shown that by using the weight function x, we reduce all proofs regarding the validity of expansion () to those for the Fourier series.

Finally, for the class of sweep signals, the following expression holds true:

$$f(t) = \frac{1}{2\pi}\int_0^{\infty}\frac{F(\alpha)}{t} * e^{-i\alpha t^2}d\alpha; \quad (19)$$

Где $F(\alpha) = \frac{1}{2\pi}\int_0^{\infty} t * f(t) * e^{-i\alpha t^2}dt;$ (20)

Expression (20) is the desired Kaptz-spectrum or frequency growth rate spectrum, similar to a partial spectrum.

Let us consider transformation (20) for a sweep signal: f(t)=sin $\alpha_0(t+t_H)^2$, which has two unknown parameters:

1. $\alpha_0 = \frac{\omega_H - \omega_K}{2T}$ – Frequency growth rate;

$|\omega_H - \omega_K|$ – Frequency range,

T – Signal duration,

2. t_H – Initial time (or arrival time).

As a result of the calculations, we obtain the real and imaginary parts of the Kaptz-spectrum in the form:

$$F_{Re}(\alpha) = \frac{1}{2\pi}\int_0^{t_0} t * sin\alpha_0(t+t_H)^2 * cos\alpha * t^2 dt = \frac{1}{2\pi} * \frac{cos\left((\alpha-\alpha_0)*\left(T+\frac{\alpha_0 t_H^2}{\alpha-\alpha_0}\right)^2 - \frac{\alpha\alpha_0 t_H^2}{\alpha-\alpha_0}\right)}{\alpha-\alpha_0+\alpha_0\frac{t_H}{T}}; \quad (21)$$

$$F_{lm}(\alpha) = \frac{1}{2\pi}\int_0^{t} t * sin\alpha_0(t+t_H)^2 * sin\alpha * t^2 dt = \frac{1}{2\pi} * \frac{sin\left((\alpha-\alpha_0)*\left(T+\frac{\alpha_0 t_H^2}{\alpha-\alpha_0}\right)^2 - \frac{\alpha\alpha_0 t_H^2}{\alpha-\alpha_0}\right)}{\alpha-\alpha_0+\alpha_0\frac{t_H}{T}}; \quad (22)$$

The frequency growth rate spectrum is defined, similarly to the frequency spectrum, by the sum of the squares of part (21) and the imaginary part (22) – let us call it the Kaptz-spectrum:

$$|F(\alpha)| = F_{Re}^2(\alpha) + F_{lm}^2(\alpha) = \frac{1}{8\pi}\frac{1}{\left(\alpha-\alpha_0+\alpha_0\frac{t_H}{T_0}\right)^2}; \quad (23)$$

The maximum of the spectrum corresponds to the value: $\alpha = \alpha_0 * (1 + \frac{t_H}{T})$;

It can be seen that the Kaptz-spectrum depends on two parameters: the initial time t_H and the frequency growth rate α_0. Therefore, if either parameter is known, the other can be determined. The maximum of the Kaptz-spectrum corresponds to the value:

$\alpha = \alpha_0 + \alpha_0 \frac{t_H}{T_0}$, i.e., if t_H- is known, then α_0 is determined, and vice versa.

The discretization step in the Kaptz domain will be equal to: $\Delta\alpha = \frac{1}{T^2}$;

The transformation of a sweep signal with an arbitrary initial frequency will take the following form:

$$Re \int_0^T t\, sin(\alpha_0 t^2 + 2\alpha_0 t_1 t + \varphi_1) cos(\alpha t^2 + 2\alpha t_2 t) dt = \left\{(\alpha_0 - \alpha)\left[\left(t + \frac{\Delta\varphi}{\Delta\alpha}\right)^2 - \left(\frac{\Delta\varphi}{\Delta\alpha}\right)^2 + \frac{\varphi_1}{\Delta\alpha}\right]\right\} =$$

$$= |\Delta\alpha = \alpha_0 - \alpha\ \Delta\varphi = \alpha_0 t_1 - \alpha t_2\ | = \frac{1}{4}\frac{T}{\Delta\alpha\left(T + \frac{\Delta\varphi}{\Delta\alpha}\right)} cos\left(\Delta\alpha\left(T + \frac{\Delta\varphi}{\Delta\alpha}\right)^2 + \varphi\right) - \frac{1}{4}\int_0^T \frac{\Delta\varphi}{\Delta\alpha}\frac{cos\,(*)^2}{(*)} d*$$

$$Re \approx \frac{1}{4}\frac{T}{\Delta\alpha T + \Delta\varphi} cos\left(\Delta\alpha\left(T + \frac{\Delta\varphi}{\Delta\alpha}\right)^2 + \varphi\right); \qquad \Delta\varphi = \alpha_0 t_1 - \alpha t_2$$

$$Lm = \frac{1}{4}\frac{T}{\Delta T + \Delta\varphi} sin\left(\Delta\alpha\left(T + \frac{\Delta\varphi}{\Delta\alpha}\right)^2 + \varphi\right); \qquad \Delta\alpha = \alpha_0 - \alpha$$

$$|F| = \sqrt{Re^2 + Lm^2} = \frac{1}{2}\frac{T}{\Delta\alpha T + \alpha_0 t_1 - \alpha t_2} = \frac{1}{4}\frac{T}{(\alpha_0 T + \alpha_0 t_1 - \alpha T - \alpha t_2)};$$

Condition for the maximum

$$\frac{\alpha_0 T + \alpha_0 t_1 - \alpha(T + t_2)}{T} = 0;$$

$$\alpha\left(1 + \frac{t_1}{T}\right) = \alpha\left(1 + \frac{t_2}{T}\right)$$

The maximum of the α-spectrum:

$\alpha = \alpha_0 \frac{\left(1+\frac{t_1}{T}\right)}{\left(1+\frac{t_2}{T}\right)}$; t_1 is the arrival time of the sweep signal, t_2 is the arrival time of the reference sweep signal.

Applications of the transformation:

1. Identification of variable-frequency patterns – Kaptz-regularities.

2. Modeling of consciousness processes based on the latest theories of its emergence (quantum theories and the theory of increasing frequencies of physiological processes in a neural hypernetwork).

3. Digitalization of social processes and the theory of subject relationships; processing and integration of analysis across all scientific fields (e.g., discretization of the chemistry of odors).

4. Signal extraction in vibroseismology: determination of the sweep signal frequency growth rate and the sweep signal arrival time.

5. Exploiting the invertibility of the transformation to perform filtering in the Kaptz domain for enhanced signal-to-noise ratio.

6. Utilization of the transformation in voice and pattern recognition as one of the most stable parameters.

7. Physical example: two particles propagate and oscillate within identical spectra, yet possess different energies and properties. One of the underlying causes may be a varying frequency growth rate.

In image interference patterns – holograms.

Also in particle quantization, where a specific time interval exists depending on the frequency growth rate, and the energy (the square of the spectrum over this interval) does not increase when the analysis time interval is increased.

8. For a deeper study of the correlation of phenomena – alongside the frequency spectrum – comparing Kaptz-spectra allows for more precise determination of genetic and phenotypic similarities, for instance.

9. Algorithms for trading software/bots on stock exchanges; forecasting processes.

10. In the theory of subject relationships, the Kaptz unit serves as a metric for relationships – i.e., as soon as the relationship changes by one Kaptz, it signifies a shift in frequency within a specific mode of interaction (visual, tactile, mental, physical, etc.). The rate of change in

relationships is measured in Kaptz. The essence of the concept of relationships is the periodic interaction of subjects over a certain period of time at a specific frequency for defined purposes.

11. The prefix "Kaptz" in linguistics also modifies the essence of words when the existing meaning of a word is no longer precise enough to describe a phenomenon. To "Kaptzify" a phenomenon means to decompose the described process into time intervals, periods, frequencies, and forms of interaction.

The use of Kaptz-conversion is protected by copyright. Consultations and programming of applied algorithms are available in accordance with development requests.

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